Eur. Phys. J. A **4**, 291–305 (1999)

THE EUROPEAN PHYSICAL JOURNAL A

Formation of correlations and energy-conservation at short times

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Received: 25 November 1998 / Revised version: 11 January 1999
Communicated by P. Povh

Abstract. The formation of correlations due to collisions in an interacting nucleonic system is investigated. Results from one-time kinetic equations are compared with the Kadanoff and Baym two-time equation with collisions included in Born approximation. A reasonable agreement is found for a proposed approximation of the memory effects by a finite duration of collisions. This form of collision integral is in agreement with intuitive estimates from Fermi's golden rule. The formation of correlations and the build up time is calculated analytically for the high temperature and the low temperature limit. Different approximate expressions are compared with the numerical results. We present analytically the time dependent interaction energy and the formation time for Gauß- and Yukawa type of potentials.

PACS. 05.20.Dd Kinetic theory – 24.10.Cn Many-body theory – 72.10.Bg General formulation of transport theory – 82.20.Mj Nonequilibrium kinetics

1 Introduction

The Boltzmann transport equation has played a very important role in the development of non-equilibrium statistical mechanics. This microscopic equation describes the time-evolution of a distribution-function in phase-space and has also provided a connection with macroscopic hydrodynamic equations by a moment expansion of the momentum. Important applications are for example the well-known Chapman-Enskog calculations of transport coefficients. In later developments the Markovian Boltzmann-equation has been extended to include memory and correlation-effects in the collision-integral and there are a large number of publications concerning such improvements. These classical kinetic equations describe the time-evolution of a one-time distribution function $f(\mathbf{r}, \mathbf{p}, t)$.

Meanwhile, a quantum two-time theory for the time-evolution of real time Green's functions $G(\mathbf{r}, \mathbf{p}, t, t')$ has been developed using the Schwinger-Keldysh formalism. The quantum image of the classical Boltzmann equation is usually referred to as the Kadanoff-Baym (KB) equations [1]. These equations have often been considered too complicated to solve numerically in the past. However, several numerical applications exist now. The Kadanoff-Baym equations have also played an important role in the improvements of the Boltzmann equation especially by using the *Generalised Kadanoff Baym Ansatz* (GKB) of Lipavsky et al [2]. This ansatz allows a reduction of the two-time formalism to a formally simpler one-time formalism e.g. the Boltzmann equation. The time off-diagonal Green's function elements are related by GKB to the time-diagonal by the spectral-functions. By various approximations of the spectral-functions various one-time approximations of the two-time equations can be obtained.(See e.g. [3])

These kinetic equations describe different relaxation stages. During the very fast first stage, correlations imposed by the initial preparation of the system are decaying [4,5]. These are contained in off-shell or dephasing processes described by two-time propagators. During this stage of relaxation the quasiparticle picture is established [6,7]. After this very fast process the second state develops during which the one-particle distribution relaxes towards the equilibrium value [8] with a relaxation time τ_{rel}. First the momentum anisotropy relaxes by small angle scattering events and then the energetic degrees of freedom relax. During this relaxation state the virial corrections are established and can be consistently described by a nonlocal Boltzmann kinetic equation [9,10]. The time of the first stage τ_c is mostly shorter than the relaxation time τ_{rel} of one particle distributions which is entirely determined by the collision process. We will focus on the first stage which is related to the formation of correlations.

The formation of correlations is connected with an increase of the kinetic energy or equivalently the build up of correlation energy. This is due to rearrangement processes which let decay higher order correlation functions until only the one - particle distribution function relaxes. Because the correlation energy is a two - particle observable we expect that the relaxation of higher order correlations can be observed best within this quantity. Of course, the total energy of the system is conserved

$$\frac{\partial}{\partial t}\left(\left\langle \frac{p_1^2}{2m}\right\rangle(t)+E_{\rm corr}(t)\right)=0, \tag{1}$$

which means that the kinetic energy increases on cost of the correlation energy $E_{\rm corr}(t)$. We will observe a transformation of correlation into kinetic energy. This process saturates on the end of the first stage of relaxation. It is more convenient to calculate the kinetic energy than the correlation energy because the kinetic energy is a one-particle observable. Consequently, the time dependence of the kinetic energy will be investigated within the kinetic theory. This can only be accomplished if we employ a kinetic equation which leads to the total energy conservation (1). It is immediately obvious that the ordinary Boltzmann equation cannot be appropriate for this purpose because the kinetic energy is in this case an invariant of the collision integral and constant in time. Imposing the conservation of the form (1) we have to consider non-Markovian kinetic equations [11], which account for the formation of two-particle correlations.

Within these kinetic equations the collision integral is an expression of the two-particle correlations. While the one-particle distribution remains almost unchanged during the first stage of relaxation, the two-particle correlations relaxes. Consequently the one- particle spectral function is changing. The latter one is responsible for the dephasing and therefore formation of correlations. Analytical expressions for the time dependence of the kinetic and correlation energy are obtained in this paper by considering explicitly this dephasing process.

We start from a kinetic equation appropriate for short time scales in Born approximation. It contains the full memory-effect but no damping i.e. no explicit width of the spectral function, because quasiparticles are not yet formed on this time scale. In Chapter II we give an overview of the gradient approximation with emphasis on energy-conservation and correlation energy. In appendix B we discuss the limit of complete collisions and the weakening of initial correlations. In appendix A we calculate the equilibrium value of the correlation energy for high and low temperature limits analytically using Gaussian- and Yukawa-type interactions.

On the very first time scale we can neglect retardation effects in the one-particle distribution function, but we have to keep into account off-shell properties of the collision integral. Therefore we use the *finite duration* approximation in Chapter III. It leads to the correct equilibrium value and is intuitively clear from Fermi's Golden rule. It is compared numerically with the KB results. These calculations also bring to the attention the correlation-time i.e. the time for build-up of correlations.

From the observation that the time-variation of the distribution-functions can be neglected in the first stage of relaxation we obtain an analytic expression for the time dependent formation of correlations. Especially we give analytical results for the formation time of correlations in a high and in a low temperature limit. Comparisons are made with numerical calculations.

Chapter IV summarizes our results and we discuss some aspects regarding the correlation time. The appendices show some important relations necessary for our analytic calculations.

2 Correlation Energy in gradient expansion

The kinetic equation in Born approximation for spatial homogeneous media including complete time convolution (memory effect) but no damping is called Levinson equation and reads [12–15]

$$\begin{aligned}
&\frac{\partial}{\partial t}f(p_1)=\\
&\frac{2s_1s_2}{\hbar^2}\int\frac{dp_2dp_1'dp_2'}{(2\pi\hbar)^6}V(\mid p_1-p_1'\mid)^2\delta(p_1+p_2-p_1'-p_2')\\
&\times\int_{t_0}^{t}d\tau\cos\left(\frac{1}{\hbar}(E_1+E_2-E_1'-E_2')(t-\tau)\right)\\
&\times\left(f(p_1',\tau)f(p_2',\tau)\bar f(p_1,\tau)\bar f(p_2,\tau)\right.\\
&\left.-\,f(p_1,\tau)f(p_2,\tau)\bar f(p_1',\tau)\bar f(p_2',\tau)\right)
\end{aligned} \tag{2}$$

with $\bar f=1-f$, the free particle dispersion $E=p^2/2m$ and the spin-isospin degeneracy s_1,s_2. The distribution functions are normalized to the density as $s\int\frac{dp}{(2\pi\hbar)^3}f(p)=n$. For the sake of simplicity we have omitted the Hartree and Fock contribution. Since we discuss the correlation energy the meanfield contribution is just additive. The dispersion $E(p)$ in the collision integral is modified by the Fock term, but we use this effect only in an approximative way by understanding m as effective mass.

The Boltzmann collision integral is obtained from equation (2) if: (i) One neglects the time retardation in the distribution functions, i.e. the memory effects and (ii) The finite initial time t_0 is set equal to $-\infty$ corresponding to what is usually referred to as the limit of complete collisions. The memory effect is condensed in the explicit retardation of the distribution function. This would lead to gradient contributions to the kinetic equation which can be shown to be responsible for the formation of high energetic tails in the distribution function [16,17]. This effect will be established on the second stage of relaxation.

The second effect is contained in the energy broadening or off-shell behavior in (2). This is exclusively related to the spectral properties of the one-particle propagator and therefore determined by the relaxation of two-particle correlation. Since we are studying the very short time region after the initial disturbance we can separate the one-particle and two-particle relaxation. On this time scale the memory in the distribution functions can be neglected but we will keep the spectral relaxation implicit in the off-shell cos-function of (2). This effect is the most relevant one for obtaining the time evolution of the interaction (or correlation) energy and therefore energy conservation.

In the following discussion we shall only be concerned with the time integration. Therefore we introduce the short hand notation of (2)

$$\frac{\partial}{\partial t} f(p_1) = \frac{1}{\hbar^2} \int_{t_0}^{t} d\tau \cos \frac{\Delta E(t-\tau)}{\hbar}\, F(\tau), \qquad (3)$$

where

$$\begin{aligned} F(\tau) = 2V(\mid p_1 - p_1' \mid)^2 \delta(p_1 + p_2 - p_1' - p_2') \\ \times (f(p_1',\tau) f(p_2',\tau) \bar{f}(p_1,\tau) \bar{f}(p_2,\tau) \\ - f(p_1,\tau) f(p_2,\tau) \bar{f}(p_1',\tau) \bar{f}(p_2',\tau)) \end{aligned} \qquad (4)$$

and the 9-dimensional momentum-integration is suppressed in (3) and in the following.

From this equation one derives balance equations by integration over momentum p_1. The first two moments, i.e. the density and total linear momentum, are conserved. For the Markovian Boltzmann equation the kinetic energy is conserved, while the potential energy is zero. In the present case including the memory effect one finds [11]

$$\frac{\partial}{\partial t}(\langle \frac{p_1^2}{2m} \rangle + E_{\rm corr}) = 0, \qquad (5)$$

where the correlation energy $E_{\rm corr}$ is given by

$$\begin{aligned} & E_{\rm corr}(t) - E_{\rm corr}(t_0) \\ & \quad = -\frac{1}{4\hbar} \left\langle \int_0^{t-t_0} d\tau \, \sin\left(\frac{\Delta E \tau}{\hbar}\right) F(t-\tau) \right\rangle. \end{aligned} \qquad (6)$$

Here $\langle\rangle$ indicates the integration over p_1. We like to point out that we have neglected initial correlations in the kinetic (2) in agreement with the studied sudden switching approximation. Consequently, $E_{\rm corr}(t_0)$ describes only possible constant background correlations not formed by the binary collisions. Expanding $F(t-\tau)$ around t one obtains a gradient expansion series for the interaction energy that reads

$$\begin{aligned} E_{\rm corr}(t) - E_{\rm corr}(0) &= \sum_{n=0}^{\infty} \langle V_n(t) F^{(n)}(t) \rangle \\ V_n(t) &= -\frac{1}{4\hbar} \frac{(-1)^n}{n!} \int_0^{t-t_0} dt' t'^n \sin \frac{\Delta E t'}{\hbar}, \end{aligned} \qquad (7)$$

where the n-th time derivative of $F(t)$ is given by $F^{(n)}(t) = \frac{\partial^n}{\partial t^n} F(t)$.

Taking the time-derivative of (7) one finds

$$\frac{\partial}{\partial t} E_{\rm corr} = \sum_{n=0}^{\infty} \langle V_n'(t) F^{(n)}(t) + V_n(t) F^{(n+1)}(t) \rangle. \qquad (8)$$

On the other hand one can express the time derivative of the interaction energy in terms of a gradient expansion of the collision integral directly from Eq. (2). This leads to

$$\begin{aligned} \frac{\partial}{\partial t} E_{\rm corr} &= \sum_{n=0}^{\infty} \langle I_n(t) F^{(n)}(t) \rangle \\ I_n(t) &= -\frac{1}{4\hbar} \frac{(-1)^n}{n!} \int_0^{t-t_0} dt' t'^n \Delta E \cos \frac{\Delta E t'}{\hbar}. \end{aligned} \qquad (9)$$

Note the difference between the two expansions (9) and (8). For example, the zero order term in (8) does contain not only the zero order term but also part of the first order term of (9). This is understandable, because the collision integral determines the *time derivative* (9) of the correlation energy. One has to expand the collision integral one step further in order to obtain the correlation energy (7) up to a specific level of gradient expansion. This is a quite general observation for any order of gradient approximation.

Comparing the two gradient expansions (9) and (8) we establish a relation between $I_n(t)$ and $V_n(t)$

$$\begin{aligned} I_n &= \frac{\partial}{\partial t} V_n + V_{n-1} \\ I_0 &= \frac{\partial}{\partial t} V_0 = -\frac{1}{4\hbar} \sin \frac{\Delta E(t-t_0)}{\hbar}, \end{aligned} \qquad (10)$$

or inversely

$$\begin{aligned} V_n(t) &= \int_{t_0}^{t} dt' (I_n(t') - V_{n-1}(t')) \\ V_0(t) &= \int_{t_0}^{t} dt' I_0(t') = \frac{1}{4} \frac{\cos \frac{\Delta E(t-t_0)}{\hbar} - 1}{\Delta E}. \end{aligned} \qquad (11)$$

The long time limit of the different gradient approximations of the kinetic equation are presented in Appendix B and is found to be unique. The limit of complete collisions $t_0 \to -\infty$ and the connected problem of weakening of initial correlations are discussed there.

3 The formation of correlations

To lowest order the gradient expansion (7), the correlation energy is

$$E_{\rm corr}(t) - E_{\rm corr}(0) = \frac{1}{4} \left\langle \frac{\cos \frac{\Delta E(t-t_0)}{\hbar} - 1}{\Delta E} F^{(0)}(t) \right\rangle. \qquad (12)$$

Retaining only the first term for this correlation energy is equivalent to an approximation of the non-Markovian collision integral (2) where we neglect the time dependence of the distribution functions while keeping the finite initial time t_0. This approximation gives instead of (3)

$$
\begin{aligned}
&\frac{\partial}{\partial t} f(p_1) = \frac{1}{\hbar} \frac{\sin \frac{\Delta E(t-t_0)}{\hbar}}{\Delta E} \; F(t) \\
&= \frac{2}{\hbar} \int \frac{dp_2 dp_1' dp_2'}{(2\pi\hbar)^6} V(\mid p_1 - p_1' \mid)^2 \delta(p_1 + p_2 - p_1' - p_2') \\
&\times \frac{\sin{(E_1 + E_2 - E_1' - E_2')(t-t_0)/\hbar}}{E_1 + E_2 - E_1' - E_2'} \\
&\times [f(p_1',t) f(p_2',t) \bar{f}(p_1,t) \bar{f}(p_2,t) \\
&- f(p_1,t) f(p_2,t) \bar{f}(p_1',t) \bar{f}(p_2',t)]. \qquad (13)
\end{aligned}
$$

Using the same steps which were used to derive the correlation energy $E_{\text{corr}}(t)$ in (6) from the collision integral (2) one easily finds that the collision integral (13) gives the lowest order term of the time derivative of this correlation energy, i.e. (12). The off-shell function in this collision integral contains a memory of the initial state at t_0. This induces a memory in the kinetic equation in spite of the fact that the collision integral is formally Markovian. From (7) one recognizes that the equilibrium or long time limit of the correlation energy is exactly give by the first order gradient approximation (12) since all next orders include time derivatives and vanishes therefore on the long time scale.

We summarize three important features of this approximate collision integral: (i) It reproduces the zero order gradient term of the time dependent correlation energy (7). (ii) It leads to the complete expression for the equilibrium correlation energy (32) and (iii) it is a direct consequence of Fermi's Golden Rule, where we keep time dependent perturbation theory. Previously, this approximation has been considered also for electron plasmas [5,7,18,19].

It was shown in appendix B that the two operations of taking the time-derivative and the long-time limit of t_0 do not commute. This is nicely illustrated by Eq. (13) which gives the correct correlation energy (32) if the limit $t_0 \to -\infty$ is performed afterwards. If however the limit of complete collision $t_0 \to -\infty$ (or $t \to \infty$) is performed first on the kinetic (13) it reduces to the Markovian Boltzmann equation without correlation energy.

We refer to (13) as the *finite duration* time approximation. It carries the most important features of the build up of correlations after the interactions are switched on in the initially uncorrelated system. This will be demonstrated by some numerical examples below. We shall first present some analytical results for high and low temperature limit and compare them with the numerical solution later. To this end we assume a system consisting of two different types of particles a, b with different masses m_a, m_b. As a first example we shall consider a Yukawa-type potential of the form

$$V_{\text{Y}}(r) = \frac{g_{ab}}{r} \mathrm{e}^{-\kappa r} \qquad (14)$$

where in nuclear physics applications g_{ab} is the coupling and κ the effective range of potential given by the inverse mass of interchanging mesons. In plasma physics applications the potential (14) represents the Debye potential with $g_{ab} = e_a e_b/\epsilon_0$ given by the two charges and the inverse screening length

$$\kappa^2 = \sum_a \frac{4\pi e_a^2}{\epsilon_0} \frac{\partial n_a}{\partial \mu_a} \qquad (15)$$

where n_a is the density and μ_a the chemical potential of specie a.

As a second example we shall use a Gauß-type potential

$$V_G(r) = V_0 \mathrm{e}^{-(r/\eta)^2} \qquad (16)$$

which has been used in nuclear physics applications [20, 21] with $\eta = 0.57\text{fm}^{-3}$ and $V_0 = -453$ MeV.

We will proceed and derive analytical expressions which are compared with the numerical solution of (12). The numerical values are compared in table 1 with the solution of the KB equations. It is found an overall good agreement.

3.1 Time dependent correlation energy

We calculate the build up time of correlation, τ_c by inspecting the time derivative of the interaction energy. We define τ_c as the time at which this derivative becomes sufficiently small. This corresponds to using (9) instead of (7), but only with the first term according to the finite duration approximation of the last chapter. We have from (10)

$$\frac{\partial}{\partial t} E_{\text{corr}} = -\frac{1}{4\hbar} \left\langle \sin \frac{\Delta E(t-t_0)}{\hbar} F(t) \right\rangle . \qquad (17)$$

3.1.1 High temperature limit

In the limit of high temperature we neglect the degeneracy and the equilibrium distribution takes the Maxwell-Boltzmann form

$$f(p) = \frac{n}{s} \lambda^3 \mathrm{e}^{-\frac{p^2}{2mT}}; \qquad \lambda^2 = \frac{2\pi\hbar^2}{mT}. \qquad (18)$$

We get for the Gauß potential and $b^2 = \hbar^2/(2\mu T \eta^2)$ the result

$$
\begin{aligned}
\frac{\partial}{\partial t} E^G_{\text{corr}} &= \sum_{ab} \frac{4 n_a n_b \pi \eta^2 V_0^2}{\sqrt{2\mu T} b^4} \left(\frac{4\mu}{M}\right)^2 \\
&\times \frac{\partial}{\partial \beta} \left(\frac{\pi(\sqrt{\beta^2 + 16 t^2 T^2/\hbar^2} - \beta)}{8(\beta^2 + 16 t^2 T^2/\hbar^2)} \right)^{1/2}_{\beta = 2/b^2 + 4t^2T^2/\hbar^2} \\
&= -\sum_{ab} \frac{3 n_a n_b \pi^{3/2} \eta^6 V_0^2 (2\mu)^{3/2}}{16 T^{5/2}} \left(\frac{4\mu}{M}\right)^2 \frac{1}{t^4} + o(t^{-5}).
\end{aligned}
\qquad (19)
$$

We see a monotonic decrease of the time derivative or equivalently a monotonic increase of the correlation energy to its equilibrium value. It is remarkable that the long time limit is entirely determined by the classical value. Obviously no quantum effects enter the formation of correlation in the high temperature limit.

We can also calculate the time dependent formation for Yukawa-like potentials. The time derivative of the correlation energy leads to [appendix (63)]

$$\frac{\partial}{\partial t}\frac{E^{Y}_{\rm corr}(t)}{n} = -\frac{e^2\kappa T}{2\hbar}{\rm Im}\left[(1+2z^2){\rm e}^{z^2}(1-{\rm erf}(z)) - \frac{2z}{\sqrt{\pi}}\right] \tag{20}$$

where we used $z = \omega_p\sqrt{t^2 - it\frac{\hbar}{T}}$ and the collective (plasma) frequency $\omega_p^2 = \kappa^2 T/m$, compare with (15). This is the analytical quantum result of the time derivative of the formation of correlation. For the classical limit we are able to integrate expression (20) with respect to times and arrive at [7]

$$E^{Y}_{\rm corr}(t) - E^{Y}_{\rm corr}(0) = -\frac{n\kappa e^2}{4} \times \left[1 + \frac{2\omega_p t}{\sqrt{\pi}} - (1+2(\omega_p t)^2){\rm e}^{(\omega_p t)^2}(1-{\rm erf}(\omega_p t))\right]. \tag{21}$$

It shows that the formation of correlations is basically given by the inverse collective frequency ω_p. Therefore during the first stage of relaxation the fluctuating and collective effects are more important than collisions.

3.1.2 Low temperature limit

The low temperature value is of special interest, because it leads to a natural definition of the build up of correlations. Using the same steps as in Appendix (A.2) one obtains from (17)

$$\frac{\partial}{\partial t}E_{\rm corr} = -\frac{1}{2}m^4 p_f \left\langle \frac{V^2}{\cos\frac{\theta}{2}} \right\rangle \tilde{I}_f \tag{22}$$

with abbreviation as in (41). It is now easy to perform the time integral in (22) to obtain the time dependence of the correlation energy

$$E^{low}_{\rm corr}(t) - E^{low}_{\rm corr}(0) = E^{\rm low}_{\rm corr}\left(1 + \frac{1}{3}\left(\frac{\epsilon_f+\epsilon_c}{\pi T}\right)^2\right)^{-1} \times \left\{1 - \frac{1}{x}\sin(x) + \left(\frac{\epsilon_f+\epsilon_c}{\pi T}\right)^2\left(\frac{1}{3} + \left[\frac{1}{x}\sin(x)\right]''\right)\right\} \tag{23}$$

with $x = 2\frac{\epsilon_f+\epsilon_c}{\hbar}t$ and the equilibrium correlation energy $E^{\rm low}_{\rm corr}$ from (42) respectively. This shows that the correlation energy is built up and oscillates around the equilibrium value. This oscillations are damped with t^{-1} in time.

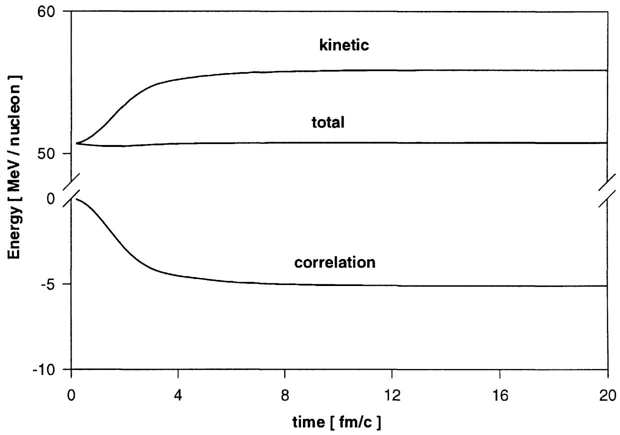

Fig. 1. The time dependent kinetic and correlation energy vs. time for a counter-flowing streams of nuclear matter from (13). The temperatures and densities of the colliding beams are $T_1 = 10$ MeV, $n_1 = n_o/60$ and $T_2 = 5$ MeV $n_2 = n_o/10$ and the relative momentum $1.5\hbar/fm$, which corresponds a colliding energy of 45 MeV/nucleon. The beams are starting to interact at time point t_0. One sees the build up of correlation energy during a correlation-time $\tau_c = 3fm/c$. The parameter are chosen in such a way that the system is degenerate, which is described by $n\lambda^3 = 0.416 < 1$. The total initial kinetic energy corresponds (neglecting correlation energy) to an equilibrated system with a temperature of $T = 32$ MeV

We would like to point out here that the result for the formation of correlations at low temperatures (23) is independent of the interaction used. Because higher order interaction described by higher order diagrams can be cast into a Boltzmann- like collision integral with more involved transition matrix elements we have always the same time dependence of (23) for binary interactions however with different equilibrium correlation energy $E_{\rm corr}$.

3.2 Numerical results

Figure 1 shows the time development of the kinetic energy as well as the correlation energy. The equation (13) has been solved numerically for two initially counter-flowing streams of nuclear matter, where the nucleons interact via the Gauß type of potential (16). The initial temperatures and densities of the colliding beams are $T_1 = 10$ MeV $n_1 = n_o/60$ and $T_2 = 5$ MeV $n_2 = n_o/10$ respectively. The relative momentum is $1.5\hbar/fm$, which corresponds to a colliding energy of 45 MeV/n. We observe a build up of correlations during the initial $3-4fm/c$. Total energy is conserved and the kinetic energy is increased by the same amount as the correlation energy is decreased. Similar results has been found for a different system in [5]. This is because the system is initially prepared to be uncorrelated at $t_0 = 0$. If time t_0 i.e. the time when the system is uncorrelated is shifted to $t_0 = -\infty$ i.e. infinite past, we would not observe any build up of correlations. The (13) would then in fact reduce to the Boltzmann equation.

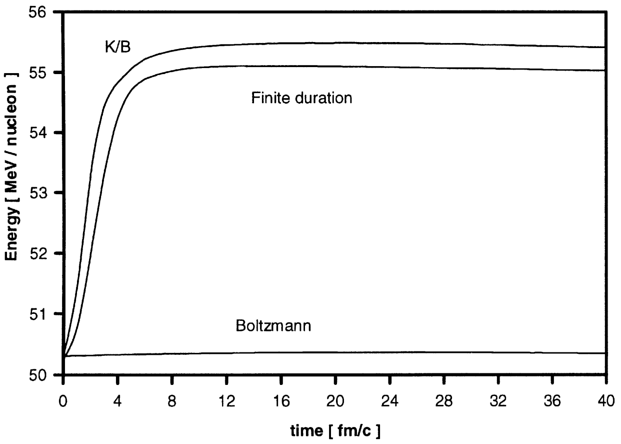

Fig. 2. The time dependent kinetic energy from a solution of the Kadanoff-Baym equation (KB) is shown together with the results from the finite duration approximation (13) and the Boltzmann equation. For Boltzmann transport the kinetic energy is conserved in each collision and therefore globally. The conclusion is that the broadening of the δ function of energy conservation in the finite duration approximation almost accounts for the time dependent built up of correlations from the exact KB equation

In Fig. 2 we compare the results with the exact solution of Kadanoff-Baym equations [21,3]. We see that the finite duration approximation reproduces the exact result quite nicely. The small deviation is due to higher order effects.

In order to investigate a situation with higher degeneracy we choose a model of two initially counter - flowing streams of nuclear matter with density and temperature $n_1 = n_o/60$ $T_1 = 0.5$ MeV and $n_2 = n_o/20$ $T_2 = 0.1$ MeV moving with relative velocity of $1\hbar/fm$ corresponding to a collision energy of 21 MeV/n. The interaction is again a Gauß-type of potential. In Fig. 3 is plotted the time evolution of the kinetic energy and the correlation.

This build up of correlations is independent of the initial distribution form. If for example we choose a (equilibrium) Fermi distribution as the initial distribution a build up of correlations will occur as well. This is due to the fact that the spatial correlations relate in momentum space to excitations, resulting in a distribution looking somewhat like a Fermi-distribution but with a temperature higher than that of the initial uncorrelated Fermi-distribution. [21]

In order to illustrate the temperature dependence of τ_c as well as to demonstrate the quality of limiting analytical formulae, we plot in Fig. 4 (thick lines) results from the solution of the Kadanoff and Baym equations for a fixed chemical potential of 37.1 MeV and for three different temperatures. The figure shows the increase of the kinetic energy (equivalent to the decrease of correlation energy) with time. The KB results are compared with those from approximation (23). One sees that initially while correlations are built up the agreement is good. Especially at low temperatures the oscillations discussed above are obvious in the approximate results while the KB calculations only show a slight overshoot at the lowest temperatures.

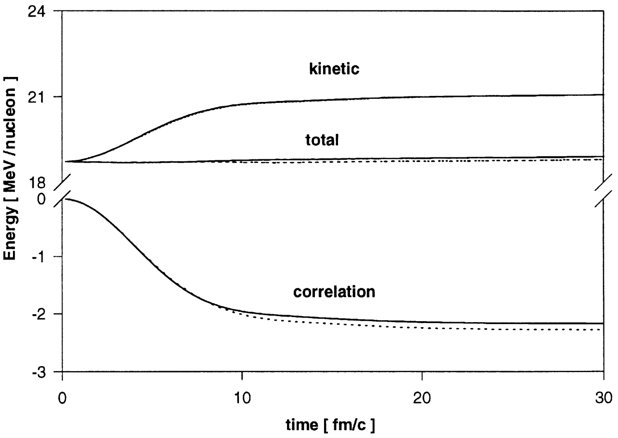

Fig. 3. The time evolution of the correlation and kinetic energy for two initially counter -flowing streams of nuclear matter with density and temperature $n_1 = n_o/60$ $T_1 = 0.5$ MeV and $n_2 = n_o/20$ $T_2 = 0.1$ MeV moving with relative velocity of $1\hbar/fm$ which corresponds to a collision energy of 21 MeV/n. In this case the correlation time $\tau_c = 9fm/c$ which is appreciably larger than in Fig. 1. We ascribe this to the smaller thermal velocity of the particles in the present case. The parameters are here in contrast to Fig. 1 chosen such that the system is degenerate $n\lambda^3 = 1.14 \geq 1$. The equilibrated temperature is here (neglecting correlations) $T = 11.6$ MeV

We believe that the discrepancy is due to the neglected damping (and perhaps due to the necessary approximations used in the integrations as discussed above). The opposite approximate formula for high temperatures (19) is plotted also for the $T = 40$ MeV case. The built up of correlations is too fast according this formula.

From the numerical inspection of this section we summarize three facts: (i) the build up of correlations is monotonic and reaches the final value smoothly, (ii) the finite duration approximation (13) is an excellent approximation and (iii) also for initial Fermi distributions we see the same build up of correlations. The latter point shows that this build up of correlation is not due to an equilibration of an initial nonequilibrium distribution but mainly due to the decay of higher order correlation function which are condensed in the off-shell *sin* - factor in the collision integral (2).

3.3 Correlation time

From the numerical observation we conclude that the correlation energy (12) is increasing monotonously with time until it reaches its almost final value (32). We assume that the main formation time of correlations $\tau = t_c - t_0$ is given by reaching this asymptotic limit. This time can be estimated by the condition

$$E_{\rm corr}(\tau) = \frac{1}{4}\left\langle \frac{\cos\frac{\Delta E\tau}{\hbar} - 1}{\Delta E} F(t_c) \right\rangle$$
$$\approx \frac{1}{4}\left\langle \frac{1}{\Delta E} F(\infty) \right\rangle \equiv E_{\rm corr}. \qquad (24)$$

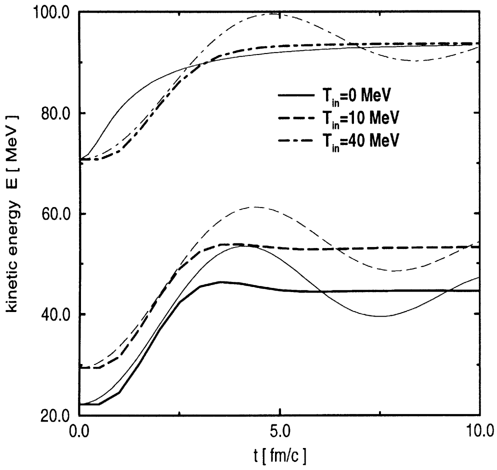

Fig. 4. The formation of correlation plotted as an increase of the kinetic energy with time for temperatures $1, 10, 40$ MeV. The chemical potential is fixed to 37.1 MeV which corresponds to densities $0.16, 0.18, 0.35 \mathrm{fm}^{-3}$. The thick lines show results from KB calculations while the thin lines are approximate values via formula (23). The equilibrium correlation energy was chosen to be equal to the KB result. The oscillations are overestimated by the approximate formula. For $T = 40$ MeV we plotted also the high temperature approximate value via (19) as thin solid line. The built up of correlations is too fast

We solve this equation for τ approximately by replacing the cos-function by a linear approximation within the build up time interval $t - t_0 = (0, \pi\hbar/\Delta E)$

$$\frac{1 - \cos\frac{\Delta E(t-t_0)}{\hbar}}{\Delta E} \approx \frac{2(t-t_0)}{\pi\hbar}\mathrm{Sgn}(\Delta E). \tag{25}$$

This approximation is correct in three points, the initial time, the final formation time where $\langle V(\tau)\rangle$ has its maximum and a time point half of this maximal time at $\pi\hbar/2/\Delta E$. The latter point coincides too because $1 - \cos xt$ has there a turning point. Therefore this linear approximation overestimates the function in the first half of the interval $t - t_0 = (0, \pi\hbar/\Delta E)$ and underestimates it in the second part. Furthermore we use equilibrium distribution functions. With the help of this approximation we can solve (24)

$$\tau \approx \frac{1}{2}\pi\hbar\frac{\langle\frac{1}{\Delta E}F(\infty)\rangle}{\langle\mathrm{Sgn}(\Delta E)F(\infty)\rangle}. \tag{26}$$

If we use $\langle\mathrm{Sgn}(\Delta E)F\rangle \approx E_{\mathrm{corr}}\langle\Delta E\rangle$, where $\langle\Delta E\rangle$ is the mean transition energy of the collision, we obtain $\tau \approx \frac{\hbar}{\langle\Delta E\rangle}$. This gives the intuitive picture of an *uncertainty* principle, i.e. a smallest time scale determined by $\hbar$ divided by the transition energy.

The high temperature value of (26) can be calculated analogously to Appendix A.1. Instead of (34) and (38) we have now

$$\frac{\pi\hbar}{2\tau} = 4T\frac{\int\limits_0^\infty dx x^2 V^2(x) \int\limits_{-\infty}^{\infty} ds\,\mathrm{Sgn}(x(s+x))\mathrm{e}^{-s^2}}{\int\limits_0^\infty dx x V^2(x) \int\limits_{-\infty}^{\infty} ds\frac{\mathrm{e}^{-s^2}}{x+s}}, \tag{27}$$

with $V(x) = 1/(x^2 + b^2)$ for Yukawa potential and $V(x) = \exp -(x/b)^2$ for Gauß potential. For the latter we obtain with the help of appendix C [(62)]

$$\begin{aligned}\tau &= \frac{\pi^2\hbar}{4T}\frac{1}{(2+b^2)(\frac{\pi}{2} + \frac{\sqrt{2}b}{2+b^2} - \arctan(\sqrt{2}/b))} \\ &= \frac{\pi^2\eta}{8v_{th}}(1 - \frac{b^2}{6} + o(\hbar^4))\end{aligned} \tag{28}$$

with the thermal velocity $v_{th}^2 = T/\mu$ and $b = \hbar/\eta/\sqrt{2\mu T}$.

We see that in the low density or quasi-classical limit $b \to 0$ the formation time of correlations are determined entirely by the range of potential η divided by the thermal velocity. This result is intuitively appealing.

The opposite limit of low temperatures can also be evaluated like in Appendix A.2. This limit is independent of the used interaction because following (40) the interaction part cancels out in (26). Then we end up with Fermi integrals which are similar to (68) and the resulting formation time of correlations is via (26)

$$\tau_{\mathrm{lowT}} = \frac{\pi\hbar}{3\epsilon_f + \epsilon_c}\left(1 + (\frac{\pi T}{\epsilon_f + \epsilon_c})^2\right). \tag{29}$$

This time agrees with the time where the correlation energy (23) has reached its first maximum

$$\tau_c \approx \frac{2\hbar}{\epsilon + \epsilon_c}. \tag{30}$$

This correlation time limits the validity of quasiparticle picture which is established at times greater than this [7]. Incidentally, in the early 1950s the criterion $\hbar/k_B T < \tau$ was supposed to limit the validity of the Landau Fermi-liquid theory for metals [22]. Later it was shown by Landau that this criterion is irrelevant and he proposed the correct criterion $\tau > \hbar/\epsilon_F$. Here we have explicitly calculated the formation time of correlations which is found to be equivalent to the memory time. It is to remark that this result describes just the memory- or collision duration time $\tau_{mem} \equiv 1/E$, see [16]. For nuclear matter at saturation density this time is about $4 - 5$ fm/c and agrees with the numerical result of memory time [23].

3.4 Range of validity

The validity of the low and high temperature expressions can be discussed with the help of two parameters, the value of the degeneracy $n\lambda^3 = \exp\mu/T$ and the ratio ϵ_f/T. There are 4 cases

$$\begin{array}{llll} case\ 1 & n\lambda^3 > 1 & T > \epsilon_f & only\ for\ s > 1 \\ case\ 2 & n\lambda^3 < 1 & T > \epsilon_f & \\ case\ 3 & n\lambda^3 < 1 & T < \epsilon_f & only\ for\ s = 1 \\ case\ 4 & n\lambda^3 > 1 & T < \epsilon_f & \end{array} \tag{31}$$

where s is the degeneracy. In Fig. (5) we plot this case for nuclear matter with $m = 938$ MeV.

It is clear that the low temperature expansion can only hold for case 4 and the high temperature expansion in case 2. For nuclear matter with $s > 1$ we see that the case 1 is not covered at all by these expansions.

Table 1. Comparison between estimates discussed in text and KB-results. The first two rows are calculated with the situation of Fig. 1 and Fig. 3. The last six rows refer to results with Fermi distributions as initial conditions. In the first three of these the density was kept fixed and the temperature was changed. In the last three the chemical potential was fixed and the temperature was changed. Temperatures are given before and after the formation of correlations. The latter temperatures are higher due to the (negative) correlations. For the correlation energy $E_{\rm corr}$ and the correlation time τ_c we compare 3 different expressions with the K/B solution. The numerical values (32) and(26) has been performed by four-dimensional principal value integration. The approximate formulae for the low temperature limit (42) and (30) are given as well as the high temperature limit (39) and (28)

	n fm^{-3}	$\mu(T), \epsilon_f$ MeV	T MeV init.	fin.	$n\lambda^3$	$E_{\rm corr}$ equil. MeV (32),(42),(39)	$E_{\rm corr}$ K/B MeV	τ_c equil. fm/c (26),(30),(28)	τ_c K/B fm/c
Fig. 1 (case 2)	0.019	—, 8.9	32.7	36.1	0.4	3.0, —, 7.0	5	3.5, —, 3.5	3.5
Fig. 3 (case 2)	0.011	—, 6.2	11.6	13.5	0.8	2.1, —, 5.3	2.5	5.6, —, 4.2	8.9
case 4	0.18	26.8, 39.8	0	25.1	6.0	24.3, 24.2,—	23.4	3.4, 3.5,—	3.4
case 4	0.18	22.0, 39.8	10	29.4	4.7	24.0, 24.0,—	23.7	3.5, 4.3,—	3.5
case 1	0.18	-11.7, 39.8	40	50.2	2.1	21.3, —,—	18.3	3.6, —,—	4.0
Fig. 4 (case 4)	0.16	24.6, 37.1	0	23.7	5.9	22.2, 21.4,—	22.4	3.6, 3.8,—	3.4
Fig. 4 (case 4)	0.18	22.1, 39.9	10	29.4	4.8	24.0, 24.1,—	23.7	3.5, 4.3,—	3.5
Fig. 4 (case 1)	0.35	24.9, 62.7	40	53.6	3.8	37.4, 45.5,—	22.4	2.8, 3.8,—	4.0

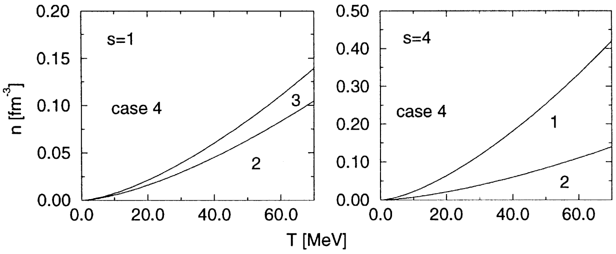

Fig. 5. The 4 different areas of density- temperature range according to (31). The high temperature expansion is applicable in case 2 and the low temperature expansion in case 4. The right figure represents the situation for spin-isospin degeneracy $s = 4$ and the left figure the situation for $s = 1$ for comparison

3.5 Discussion

We compare in Table 1 the different expansions with each other as well as with exact results from KB-calculations. The general agreement of the numerical value of the correlation energy (32) with the KB result is striking. The correlation time (26) calculated numerically agrees as well reasonably.

For the case 2 in row 1 (for a high temperature and nondegenerate system) we can reproduce the build up time quite well with the approximate formula (28). This becomes worse if we approach the degenerated condition $n\lambda^3 = 1$ in second line of the table. Both of the high temperature cases are too narrow to this condition to reproduce the correlation energy correctly. The high temperature approximate formula (39) overestimates the exact value $E_{\rm corr}$ obtained from the solution of the time dependent kinetic equation. However in the case 4 for low temperature and degenerated systems we can reproduce the simulation value quite well. The build up time is slightly overestimated by the exact low temperature value (30). The case 1 is not covered by the approximate values which is demonstrated in the 5. row of the table. There the correlation energy is much overestimated which is quite clear from the discussion of Fig 5. The systematic slight increase of the correlation time with temperature (for low temperature limit) is explained by the low temperature expansion.

The results related to Fig. 4 are represented in rows 6-8. It underlines the general good description of case 4 and the failure of case 1. The chemical potential was kept constant initially in these three cases and the temperature gradually increases, such that the chemical potential decreases. According to (30) the correlation time should increase if using the final chemical potential instead of the initial one. This is clearly not the case in K/B solutions, where the correlation time stays almost constant. This shows clearly that the formation time cannot be completely described by equilibrium formulae as done above. The correct formation time is described by a chemical potential somewhat between initial and final value.

We would like to repeat here that the expression (23) for the low temperature limit is universal for binary collision approximations, independent of the interaction. The high temperature limit should also be correct because the Born approximation used here is believed to be a good approximation for fast particles. The intermediate region is quite open. We have calculated only in Born approximation. Here especially higher order correlations beyond second Born should be employed. In summary: For low temperatures approximate analytical expressions can be given and for higher temperatures the analytical second Born approximation should be approached, while the intermediate region is left for numerical investigations.

4 Summary

The gradient approximation of the kinetic equation in second order Born approximation is investigated. The interaction energy is derived within different expansions. The equilibrium value of the correlation energy is obtained from the first order gradient expansion. This equilibrium value is calculated for Yukawa- and Gauß-type of potentials and the results are analytically given in high and low temperature limit. For contact potentials we rederive the known result for the ground state correlation energy.

A finite duration approximation of the non Markovian collision integral is proposed which follows from time dependent Fermi's Golden Rule and which is in good agreement with the numerical solution of complete collision integral. Furthermore it leads to the correct equilibrium value. Numerical comparisons are made with the solution of the complete Kadanoff and Baym equation in Born approximation and with this finite duration approximation.

The build up time of correlations is investigated and it is found that the low temperature value is universal for any approximation at the binary collision level. It is shown that the formation time of correlations is nearly determined by a ratio of $\hbar$ to the transfer energy which can be considered as an analogue to uncertainty principle.

The high temperature limit shows roughly the time scale that a particle needs to travel through the potential range. The validity of both the high and the low temperature estimates are confirmed by numerical comparisons with KB-results.

The time scale we are describing are just the life time of fireball assumed in the early stage of nuclear collision. This means the extracted temperatures from final stage products are usually wrongly extrapolated to early stages of nuclear reactions. This should effect the conclusions towards the caloric curve which is much discussed recently. Also the size and lifetime of hot reaction centers, which are extracted by interferometry methods should be critically revised as demonstrated in [24].

The authors like to thank P. Lipavský and V. Špička for interesting discussions and A. Sedrakian for helpful comments.

A Equilibrium correlation energy

From the gradient expansion (7) we see that an equilibrium value of the correlation energy $E^{\rm eq}_{\rm corr} = \langle V_0 F_0 \rangle$ is approached for large times. Here F_0 is given by Eq. (4) with equilibrium distribution functions and V_0 is given by (43) such that we have

$$\begin{aligned} E^{\rm eq}_{\rm corr}(\infty) - E^{\rm eq}_{\rm corr}(0) &= -\left\langle \frac{1}{4}\frac{P}{\Delta E} F_0 \right\rangle \\ &= -\frac{s_1 s_2}{\hbar}\int \frac{dp_1 dp_2 dp_1' dp_2'}{(2\pi\hbar)^9} V(\mid p_1 - p_1' \mid)^2 \\ &\times \delta(p_1 + p_2 - p_1' - p_2') f_0(p_1') f_0(p_2') \bar{f}_0(p_1) \bar{f}_0(p_2) \\ &\times \frac{\mathrm{P}}{E_1 + E_2 - E_1' - E_2'} \end{aligned} \qquad (32)$$

where in the second equality we have employed symmetry relations resulting in a factor 2 and have written all terms explicitly.

We are now able to calculate the correlation energy analytically. In order to compare with known results we calculate first the ground state energy for contact potentials. Then we will be able to find analytical expressions for two extreme cases, i.e. the high temperature and low temperature limit.

For performing the integrations in the collision integral we define new coordinates by $\boldsymbol{p} = \boldsymbol{p}_1' - \boldsymbol{p}_1$, $\boldsymbol{\Pi} = \frac{m_b}{(m_a+m_b)}(\boldsymbol{p}_1 + \boldsymbol{p}_1')$ and $\boldsymbol{p}' = \boldsymbol{p}_2 - \boldsymbol{p}_2'$, $\boldsymbol{\Pi}' = \frac{m_b}{(m_a+m_b)}(\boldsymbol{p}_2 + \boldsymbol{p}_2')$. For the Π, Π' coordinates we use cylindrical coordinates $\boldsymbol{\Pi} = \boldsymbol{\Pi}_\rho + \boldsymbol{\rho}$ where $\boldsymbol{\Pi}_\rho$ is parallel to the z-axis fixed by $\boldsymbol{p}$. Within these coordinates the momentum conservation takes the simple form $\boldsymbol{p}' = \boldsymbol{p}$ and the energy difference is $\Delta E = \frac{p}{2\mu}(\Pi_\rho' - \Pi_\rho)$ with μ the reduced mass.

For the contact potential we find from (32) the known ground state correlation energy [25]

$$\frac{E_{\rm corr}}{N} = \frac{3}{5}\epsilon_f \frac{4(2\log 2 - 11)}{21\pi^2}\left(\frac{p_f a}{\hbar}\right)^2 \qquad (33)$$

with the scattering length a related to the coupling constant of the contact potential $4\pi\hbar^2 a/m$. As pointed out in [26] we have to subtract an infinite value, i.e. the term proportional to $f_1 f_2$ in (32). This can be understood as renormalization of the contact potential and is formally hidden in $E^{equil}_{\rm corr}(0)$ on the left hand side. For finite range potentials we have an intrinsic cut-off due to range of interaction and such problems do not occur. For the standard procedure at low temperatures when the angular and energy integrals are separated, we have to care about this cut-off once more because during this procedure we neglect the effect of potentials but we will have to integrate off-shell.

A.1 High temperature limit

The high temperature limit is performed using the Maxwell-Boltzmann distribution (18) for the calculation of the correlation energy (32).

A.1.1 Yukawa-type potential

Performing a series of trivial integrals we obtain the expression for the correlation density for the Yukawa potential

$$\begin{aligned} E^{\rm Y}_{\rm corr} &= -\pi \sum_{ab}\left(\frac{4\mu}{m_a + m_b}\right)^2 \frac{g_{ab}^2 n_a n_b}{\kappa T} I_1[b] \\ I_1[b] &= \frac{2b}{\pi^{3/2}}\int\limits_0^\infty \frac{x dx}{(x^2+b^2)^2}\int\limits_{-\infty}^\infty \frac{ds}{s+x}{\rm e}^{-s^2} \end{aligned} \qquad (34)$$

where the abbreviation $b^2 = (\hbar\kappa)^2/8\mu T$ has been used. The parameter b proportional to $\hbar$ is responsible for quantum corrections. The last integral in (34) is performed in appendix C and the final result reads

$$E^{\rm Y}_{\rm corr} = -\pi \sum_{ab} \left(\frac{4\mu}{m_a+m_b}\right)^2 \frac{g^2_{ab} n_a n_b}{\kappa T} \times \left(1-\sqrt{\pi} b\, {\rm e}^{b^2} {\rm erfc}(b)\right). \tag{35}$$

This result is the correlation energy in second Born approximation. The quantum corrections are condensed in the b dependent function. For identical plasma particles ($g = e^2/\epsilon_0$) the result reads

$$\begin{aligned}\frac{E^{Y}_{\rm corr}}{n} &= \frac{e^2\kappa}{4\epsilon_0}\left(1-\sqrt{\pi} b\, {\rm e}^{b^2} {\rm erfc}(b)\right)\\ &= \frac{e^2\kappa}{4\epsilon_0}(1-\sqrt{\pi}b + 2b^2 + o(\hbar^3)).\end{aligned} \tag{36}$$

The classical limit shows half of the exact low density limit of correlation or self energy, [27], page 115, which we calculate in (59) in appendix C

$$\begin{aligned}E^{Debye}_{\rm corr} &= \frac{e^2\kappa}{2\epsilon_0}\frac{\sqrt{\pi}}{b}\left(1-{\rm e}^{b^2}{\rm erfc}(b)\right)\\ &= \frac{e^2\kappa}{2\epsilon_0}\left(1-\frac{\sqrt{\pi}}{2}b+\frac{2}{3}b^2+o(\hbar^3)\right).\end{aligned} \tag{37}$$

This difference follows from the static approximation of the screened interaction by the Debye potential, $W(\omega) \approx W(0) \equiv V_{\rm D}$. The energy dependency of true W reflects that the screening of Coulomb potentials is a dynamical process. The Debye potential is a correct approximation for the scattering rates, since the transferred energy ω is small being limited by the on-shell condition $\varDelta_E = 0$. For the off-shell processes, the transfered energy can reach arbitrary high values for which the Debye potential would fail. Instead we would have to use the dynamical screened potential $W(q, \frac{q(q-2k)}{2m_a})$ in the Levinson equation (2). At low energies, the imaginary part of W needed in the scattering integral reads ${\rm Im}W = |W|^2{\rm Im}\varPi \approx V_{\rm D}^2{\rm Im}\Pi$. The principle value integration in the energy function (32) can be evaluated via Kramers-Kronig relations between the real and the imaginary part of the screened potential. The conjugated real part is ${\rm Re}(W - V_{\rm C}) = V_{\rm C}{\rm Re}\varPi W \approx V_{\rm C}{\rm Re}\varPi{\rm Re}W \approx V_{\rm C}{\rm Re}\varPi V_{\rm D}$, where $\varPi$ is the susceptibility. Thus the dynamical screened potential would lead to $V_{\rm C}V_{\rm D} \sim {\rm e}^{-\kappa r}r^{-2}$, while the static approximation yields $V_{\rm D}^2 \sim {\rm e}^{-2\kappa r}r^{-2}$. The factor of two in the exponent brings the factor of two into denominator in formula (35). The corresponding integrals leading to (37) can be found in Appendix C. This means that we do not consider the formation of effective Debye potential, but use it here as an effective one present from the beginning.

A.1.2 Gauß-type potential

For the Gauß potential the same steps as before lead instead of (34) to the expression

$$E^{G}_{\rm corr} = -\frac{(\frac{4\mu}{m_a+m_b})^2\pi^{3/2}n_a n_b \eta^3 V_0^2}{2T\sqrt{2}} I_2[b],$$

$$I_2[b] = \frac{2^{3/2}}{\pi b^3}\int\limits_0^\infty x{\rm e}^{-2(x/b)^2}dx \int\limits_{-\infty}^{\infty}\frac{ds}{s+x}{\rm e}^{-s^2}, \tag{38}$$

where $b^2 = \hbar^2/(2\mu T\eta^2)$. With the help of (62) in appendix C we obtain

$$E^{G}_{\rm corr} = -\left(\frac{4\mu}{m_a+m_b}\right)^2\frac{\pi^{3/2}V_0^2\eta^3 n_a n_b}{2\sqrt{2}T}\frac{1}{2+b^2}. \tag{39}$$

A.2 Low temperature limit

In the opposite limit of small temperatures compared to the Fermi energy we employ the well known methods of Fermi liquid theory [28,29]. For simplicity we now use only one type of particles. The angular and energy integrals in (32) can then be separated with the result

$$E_{\rm corr} = -\frac{1}{2}m^4 p_f \left\langle \frac{V^2}{\cos\frac{\theta}{2}}\right\rangle I_f, \tag{40}$$

where

$$\left\langle \frac{V^2}{\cos\frac{\theta}{2}}\right\rangle = \frac{1}{(2\pi\hbar)^9}\int\frac{d\varOmega_1 d\varOmega d\phi_2}{\cos\frac{\theta}{2}}V^2(p_1-p_1') \tag{41}$$

with standard notation of angular integrals [28]. The Fermi integral I_f is done in appendix D and the final result for the Yukawa- or Gauß-potential reads

$$E_{\rm corr} = \frac{1}{6}s_1 s_2\tilde\epsilon_f\left(T^2+\frac{1}{3}\left(\frac{4\tilde\epsilon_f}{\pi}\right)^2\right)\left(\frac{m}{\hbar^2}\right)^4 \times\begin{cases}\frac{g^2}{2\kappa^3\pi^2}\left({\rm arctan}\,\frac{1}{b_l}+\frac{b_l}{1+b_l^2}\right) & Yuk.\\ \frac{V_0^2\eta^5}{2\sqrt{2\pi}}{\rm erf}(\frac{p_f\eta\sqrt{2}}{\hbar}) & Gau\ss\end{cases} \tag{42}$$

where $\tilde\epsilon_f = \frac{\epsilon_f+\epsilon_c}{4}$ and $b_l = \frac{\hbar\kappa}{2p_f}$. The best choice for cut-off we found $\epsilon_c \approx \epsilon_f$. This result shows that even for zero temperature we obtain a correlation energy due to ground state correlations by Pauli blocking, which we have explicitly demonstrated for contact potentials (33).

It should be noted however that the temperature T refers to the final equilibrated distribution. It is, in general, not a Fermi-distribution but is modified because of the correlations. This is for example illustrated by the results shown in figures 1 and 3 in [21].

A.3 Numerical values

In order to check these low and high temperature approximations, we plot in Fig. 6 the results from (42) and (39) with the numerical result of (32) for the Gauß-potential. We have chosen a normal nuclear density, $n = 0.18\,{\rm fm}^{-3}$. We see that the low temperature expansion, (42), gives

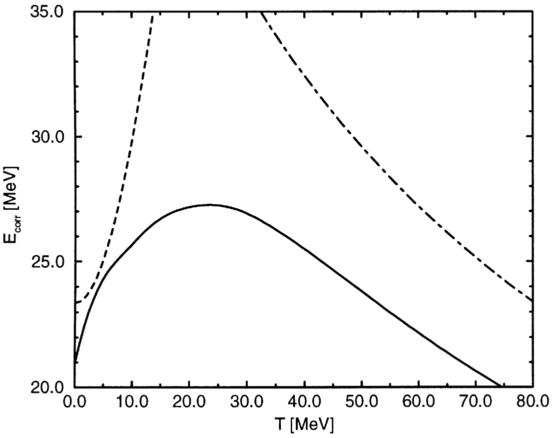

Fig. 6. The correlation energy (32), solid line, is compared with the low (42), dotted line, and the high temperature (39), dot-dashed line, approximations. Results are shown as a function of temperature at a nuclear density of $n = 0.18\mathrm{fm}^{-3}$. See also discussion in text

reasonable agreement with (32) at low temperatures only. The high temperature limit is approached actually only at very high temperatures beyond 100 MeV at this density. We would like to emphasize that the numerical computation of of (32) involves a complicated four - dimensional principal value integration. While we consider the high temperature limit to be accurate, the low temperature limit is approximate and dependent on the energy integration. The cut-off value of the energy integration (see discussion after (32)) was chosen to be ϵ_f throughout the calculation. In the low temperature limit we used instead of ϵ_f the Sommerfeld expansion $\mu(T) = \epsilon_f \left(1 - \frac{\pi^2}{12}\left(\frac{T}{\epsilon_f}\right)^2\right) + o(T^3)$.

B Weakening of initial correlation and the limit of complete collisions

In order to derive kinetic equations for the long time evolution one uses the limit $t_0 \to -\infty$ where t_0 is the initial time where correlations beyond the one-body are assumed to be absent. Two-particle correlations vanish then in the remote past. From (3) it is obvious that we could consider $t \to \infty$ alternatively. This is sometimes referred as *complete collisions* , i.e. uncorrelated asymptotic states, which can include one-body correlations, of course.

We like to show now that special care is needed when this limit is carried out, especially, if one likes to consider non-markovian corrections as we did within the gradient approximation (7). Especially we will find that the ordering of the gradient expansion and this long time limit are not easily interchangeable.

To illustrate this fact let us first carry out the $t_0 \to -\infty$ limit on the zero and first order terms $V_n(t)$ in (7). This implies that we consider the limit of complete collisions. The result is

$$\begin{aligned} V_0(t) &= -\frac{1-\cos\frac{\Delta E(t-t_0)}{\hbar}}{4\Delta E} \to -\frac{1}{4}\frac{\mathrm{P}}{\Delta E}, \\ V_1(t) &= -\frac{\hbar}{4}\frac{\partial}{\partial\,\Delta E}\frac{\sin\frac{\Delta E(t-t_0)}{\hbar}}{\Delta E} \to -\frac{\hbar}{4}\delta'(\Delta E), \\ &\dots \\ V_n(t) &\propto \hbar^n, \end{aligned} \tag{43}$$

where P is the principal value.

Therefore we obtain in the long time limit (i.e. for completed collisions)

$$\frac{\partial}{\partial\, t}\lim_{t_0\to-\infty} E_{\mathrm{corr}} = -\left\langle \frac{1}{4}\frac{\mathrm{P}}{\Delta E}\frac{\partial}{\partial\, t}F(t)\right\rangle + o(F''(t)). \tag{44}$$

On the other hand we can *first* perform the time derivative and *then* the long time limit. To this end we employ the gradient expansion of the collision integral (9) and obtain with the help of (10)

$$\begin{aligned} \frac{\partial}{\partial\, t}E_{\mathrm{corr}} &= \langle I_0 F(t)\rangle + \langle I_1 F'(t)\rangle + o(F''(t)) \\ &= -\frac{1}{4}\frac{\partial}{\partial\, t}\left\langle \left[\frac{1-\cos\frac{\Delta E(t-t_0)}{\hbar}}{\Delta E}F(t)\right]\right\rangle \\ &\quad + \left\langle \frac{1}{4\hbar}(t-t_0)\sin\frac{\Delta E(t-t_0)}{\hbar}F'(t)\right\rangle \\ \to -\frac{\partial}{\partial\, t} &\quad \left\langle \frac{1}{4}\frac{\mathrm{P}}{\Delta E}F(t)\right\rangle - \left\langle \frac{1}{4}\epsilon\frac{\partial}{\partial\,\epsilon}\frac{\mathrm{P}}{\Delta E}F'(t)\right\rangle, \end{aligned} \tag{45}$$

where the relations $T\cos xT \to -\epsilon\frac{\partial}{\partial\epsilon}\pi\delta^\epsilon(x)$ and $T\sin xT \to -\epsilon\frac{\partial}{\partial\epsilon}\frac{\mathrm{P}}{x}$ has been used. We see that we obtain with the help of $x\frac{\mathrm{P}'}{x} = -\frac{\mathrm{P}}{x} - \epsilon\frac{\partial}{\partial\epsilon}\frac{\mathrm{P}}{x}$

$$\lim_{t_0\to-\infty}\frac{\partial}{\partial\, t}E_{\mathrm{corr}} = \frac{1}{4}\left\langle \Delta E\frac{\mathrm{P}'}{\Delta E}F'(t)\right\rangle + o(F''(t)), \tag{46}$$

where P' is the derivative of the principal value. Comparing with (44) we recognize that

$$\begin{aligned} \left(\lim_{t_0\to-\infty}\frac{\partial}{\partial\, t} - \frac{\partial}{\partial\, t}\lim_{t_0\to-\infty}\right)E_{\mathrm{corr}} &= \\ -\left\langle \frac{1}{4}\epsilon\frac{\partial}{\partial\,\epsilon}\frac{P}{\Delta E}F'(t)\right\rangle. \end{aligned} \tag{47}$$

Here we can deduce the rule from (47) for large times

$$E_{\mathrm{corr}}^{\mathrm{grad-exact}}(t) >= E_{\mathrm{corr}}^{\mathrm{grad-coll}}(t) + \left\langle \frac{1}{4}\epsilon\frac{\partial}{\partial\,\epsilon}\frac{P}{\Delta E}F(t)\right\rangle \tag{48}$$

that the exact correlation energy in gradient approximation for large times $E_{\mathrm{corr}}^{\mathrm{grad-exact}}(t)$ is given by the correlation energy from gradient expansion of collision integral $E_{\mathrm{corr}}^{\mathrm{grad-coll}}(t)$ via an infinitesimal correction term. For any function F with existing derivatives the distribution $\epsilon \frac{\partial}{\partial \epsilon} \frac{P}{\Delta E}$ vanishes. Therefore we are allowed to identify the correlation energy from the kinetic equation in gradient approximation with the corresponding correlation energy in second Born approximation. Total energy is conserved in first order gradient expansion for nondegenerate systems [18]. For the degenerate systems we can prove complete energy conservation also for degenerate and space nonlocal systems [10].

However we like to point out that the above result is not valid if the transition probability contains poles, e.g. bound states or pairing ones. Then we get a nonvanishing last term in (48) expressing long living correlations such as bound states or pairing. The same discussion is applicable for formal divergent expressions as it appears e.g. for contact potentials. With the help of (48) we can renormalize the expression to obtain the ground state energy as we will demonstrate later.

B.1 Long time limit of the collision integral

The message of the last paragraph does not mean that the long time limit $t_0 \to -\infty$ of the collision integral is not uniquely defined. To show this we consider the two orderings of operations of the time derivatives and the limit of remote past respectively. For this purpose we write the gradient expansion up to first order of the kinetic (3) in two different ways

$$\begin{aligned}\frac{\partial}{\partial t} f_1 &= \frac{1}{\hbar}\frac{\sin\frac{(t-t_0)\Delta E}{\hbar}}{\Delta E}F(t)\\ &\quad+\frac{\partial}{\partial \Delta E}\frac{\cos\frac{\Delta E(t-t_0)}{\hbar}-1}{\Delta E}F'(t)+o(F''(t))\\ &= \frac{1}{\hbar}\left(\frac{\sin\frac{(t-t_0)\Delta E}{\hbar}}{\Delta E}+\frac{(t-t_0)}{\hbar}\cos\frac{\Delta E(t-t_0)}{\hbar}\right)F(t)\\ &\quad+\frac{\partial}{\partial t}\left[\frac{\partial}{\partial \Delta E}\frac{\cos\frac{\Delta E(t-t_0)}{\hbar}-1}{\Delta E}F(t)\right]. \qquad (49)\end{aligned}$$

The first equality is written in terms of the derivatives F^n. Now we take the limit $t_0 \to -\infty$ on this and obtain

$$\begin{aligned}\frac{\partial}{\partial t} f_1 &= \frac{1}{\hbar}\frac{\epsilon}{\epsilon^2+(\Delta E)^2}F(t) - \frac{\partial}{\partial t}\left[\frac{\partial}{\partial \Delta E}\frac{\Delta E}{(\Delta E)^2+\epsilon^2}F(t)\right]\\ &\equiv \frac{\pi}{\hbar}\delta(\Delta E)\,F(t) - \frac{\partial}{\partial t}\left[\frac{\mathrm{P}'}{\Delta E}\,F(t)\right]. \qquad (50)\end{aligned}$$

Equation (50) is just the generalized kinetic equation derived in [16,17] which leads to quantum Beth-Uhlenbeck virial corrections in nonequilibrium. It consists of the usual Boltzmann collision integral and a correction represented by the second term on the right hand side of (50).

We next invert the order of operations, i.e. the time derivation and the long time limit, and therefore use the second equality of (49) where the time derivative is in front of the collision integral. Applying the limit $t_0 \to -\infty$ one obtains

$$\begin{aligned}\frac{\partial}{\partial t} f_1 &= \frac{1}{\hbar}\left(\frac{\epsilon}{\epsilon^2+(\Delta E)^2}-\epsilon\frac{\partial}{\partial \epsilon}\frac{\epsilon}{\epsilon^2+(\Delta E)^2}\right)F(t)\\ &\quad-\frac{\partial}{\partial t}\left[\frac{\partial}{\partial \Delta E}\frac{\Delta E}{(\Delta E)^2+\epsilon^2}F(t)\right]. \qquad (51)\end{aligned}$$

Whereas the second term is just the same principal value expression as in (50), the first term is at first glance different, but one finds

$$\begin{aligned}\frac{\epsilon}{\epsilon^2+(\Delta E)^2}-\epsilon\frac{\partial}{\partial \epsilon}\frac{\epsilon}{\epsilon^2+(\Delta E)^2} &= \frac{\epsilon}{\epsilon^2+(\Delta E)^2}\frac{2\epsilon^2}{\epsilon^2+(\Delta E)^2}\\ &= \pi\delta(\Delta E) \qquad (52)\end{aligned}$$

so that the two results (50) and (51) are in fact equivalent.

This means that despite the care which is necessary for obtaining the mean value of the correlation energy (48), the first order gradient expansion for the generalized kinetic equation has a unique long time limit independent of the ordering of the two operations involved.

C Error function integrals

The following integral is needed in (35) for the calculation of the correlation energy

$$\begin{aligned}I_1[b] &= \frac{2b}{\pi^{3/2}}\int\limits_0^\infty \frac{x dx}{(x^2+b^2)^2}\int\limits_{-\infty}^\infty \frac{ds}{s+x}\mathrm{e}^{-s^2}\\ &= -\frac{2b}{\sqrt{\pi}}\int\limits_0^\infty \frac{x dx}{(x^2+b^2)^2}\mathrm{Im}\, w(-x+i\epsilon), \qquad (53)\end{aligned}$$

where the complex error function has been introduced [30]

$$w(z+i\epsilon) = \frac{i}{\pi}\int\limits_{-\infty}^\infty \frac{\mathrm{e}^{-t^2}}{z-t+i\epsilon}. \qquad (54)$$

Performing a partial integration and using $w'(z) = -2zw(z)+2i/\sqrt{\pi}$ we get

$$I_1[b] = 1+\frac{2b}{\sqrt{\pi}}\int\limits_0^\infty \frac{x dx}{(x^2+b^2)}\mathrm{Im}\, w(-x+i\epsilon). \qquad (55)$$

Reintroducing (54) the last integral is easily carried out by interchanging the two integrals and we obtain finally

$$I_1[b] = 1-b\sqrt{\pi}\mathrm{e}^{b^2}\mathrm{erfc}(b). \qquad (56)$$

In order to verify (37) we have instead of (53)

$$
\begin{aligned}
I_{11}[b] &= \frac{2b}{\pi^{3/2}} \int\limits_0^\infty \frac{dx}{x(x^2+b^2)} \int\limits_{-\infty}^\infty \frac{ds}{s+x} \mathrm{e}^{-s^2} \\
&= \frac{1}{b\pi^{3/2}} \left(\int\limits_{-\infty}^\infty \frac{dx}{x} \int\limits_{-\infty}^\infty \frac{ds}{s+x} \mathrm{e}^{-s^2} \right. \\
&\left. -\pi b \int\limits_{-\infty}^\infty \frac{ds}{s^2+b^2} \mathrm{e}^{-s^2} \right), \qquad (57)
\end{aligned}
$$

where a simple decomposition has been performed. For the first part we apply the Poincaré- Bertrand theorem [31] which states

$$
\begin{aligned}
&\int \frac{dx}{x-u} \int \frac{dy}{y-x} f(x,y) = \\
&\int dy \int dx \frac{f(x,y)}{(x-u)(y-x)} - \pi^2 f(u,u) \qquad (58)
\end{aligned}
$$

and obtain

$$
I_{11} = \frac{\sqrt{\pi}}{b}(1-\mathrm{e}^{b^2}\mathrm{erfc}(b)). \qquad (59)
$$

Next we perform the integral required in (38) where we put $c = 2/b^2$

$$
\begin{aligned}
\frac{\sqrt{\pi}b^3}{2^{3/2}} I_2[b] &= -\frac{1}{\sqrt{\pi}} \int\limits_0^\infty dx x \mathrm{e}^{-cx^2} \int\limits_{-\infty}^\infty \frac{ds}{s-x} \mathrm{e}^{-s^2} \\
&= -\frac{1}{\sqrt{\pi}} \int\limits_0^\infty \frac{ds}{s} \mathrm{e}^{-\frac{c}{1+c}s^2} (f(s)-f(-s)) \\
f(s) &= \int\limits_0^\infty dx x \mathrm{e}^{-(1+c)(x+\frac{s}{1+c})^2}. \qquad (60)
\end{aligned}
$$

Here we have derived the last form by dividing the integral s into two parts corresponding to the pole at $s = \pm x$ and performing variable substitution. The integral $f(s)$ is elementary and yield

$$
f(s) = -\frac{s}{(1+c)^{3/2}} \frac{\sqrt{\pi}}{2} \mathrm{erfc}\left(\frac{s}{\sqrt{1+c}} \right) + \frac{1}{2(1+c)} \mathrm{e}^{-\frac{s^2}{(1+c)}}. \qquad (61)
$$

Substituting this into (60) we obtain the result

$$
\begin{aligned}
I_2[b] &= \frac{2^{3/2}}{b^3(1+c)^{3/2}\sqrt{\pi}} \int\limits_0^\infty \mathrm{e}^{-\frac{c}{1+c}s^2} ds \\
&= \frac{\sqrt{2}}{b^3\sqrt{c}} \frac{1}{1+c} \\
&= \frac{1}{b^2+2}. \qquad (62)
\end{aligned}
$$

In (20) we need the integral

$$
\begin{aligned}
I_4[b] &= \frac{4b}{\pi^{3/2}} \int\limits_0^\infty \frac{x^2 dx}{(x^2+b^2)^2} \int\limits_{-\infty}^\infty ds \mathrm{e}^{-s^2} \sin{(s+x)xt} \\
&= \frac{2b}{\pi} \int\limits_{-\infty}^\infty \frac{x^2 \sin tx^2 dx}{(x^2+b^2)^2} \mathrm{e}^{-t^2x^2/4} \\
&= -\frac{1}{\pi} \frac{\partial}{\partial b} \mathrm{Im} \int\limits_{-\infty}^\infty \frac{x^2 dx}{(x^2+b^2)} \mathrm{e}^{-(t^2/4+it)x^2} \\
&= \mathrm{Im} \left[(1+2z^2)\mathrm{e}^{z^2}(1-\mathrm{erf}(z)) - \frac{2z}{\sqrt{\pi}} \right], \qquad (63)
\end{aligned}
$$

where we used $z = b\sqrt{t^2/4 - it}$.

Another error function integral is used in (28)

$$
I_3[a] = \sqrt{\pi} \int\limits_0^\infty dx x^2 \mathrm{e}^{-ax^2} \mathrm{erf}(x). \qquad (64)
$$

To this end we use a tabulated integral [32] (6.285.1)

$$
\begin{aligned}
I_{31}[a] &= \int\limits_0^\infty dx \mathrm{e}^{-ax^2} \mathrm{erf}(x) \\
&= \frac{\frac{\pi}{2} - \arctan\sqrt{a}}{\sqrt{\pi a}} \qquad (65)
\end{aligned}
$$

The desired integral is then

$$
\begin{aligned}
I_3[a] &= -\sqrt{\pi} \frac{\partial}{\partial a} I_{31}[a] \\
&= \frac{1}{2a^{3/2}} \left(\frac{\pi}{2} - \arctan(\sqrt{a}) + \frac{\sqrt{a}}{(1+a)} \right). \qquad (66)
\end{aligned}
$$

D Calculation of collision integral

In order to calculate the collision integral for Fermi functions in (40) special care is needed. The problem differs from the usual calculations of collision integrals in that we do not have energy conservation but an off-shell principal value. Therefore the limit $\lambda = \mu/T \to \infty$ cannot be performed straightforwardly. The reason is that the energies are restricted to the neighborhood of the Fermi level, but we have to allow off-shellness due to the denominator of the energy expression. We have used the approximation to separate the collision integral into angular and energy integrations. This leads to formally divergent results because the matrix element of interaction is not included in the energy integrals and the latter become infinite. In order to cure this defect we restrict the energies available from above. Then the expressions become analytical and we fix the upper limit of energy integration by matching to the numerical result found from (32). The best fit was found to be two times Fermi energy, i.e. $\lambda_c = \lambda$. We shortly sketch the derivation with this upper limit.

Introducing dimensional variables $x = (\epsilon - \mu)/T$ and $\lambda = \mu/T$ we have

$$I_f = T^3 \int\limits_{-\lambda}^{\lambda_C} \frac{dx_1 dx_2 dx_d x_2'}{(\mathrm{e}^{x_1}+1)(\mathrm{e}^{x_2}+1)(\mathrm{e}^{-x_1'}+1)(\mathrm{e}^{-x_2'}+1)} \times \frac{1}{(x_1+x_2-x_1'-x_2')}. \tag{67}$$

Now we switch to difference and center of mass coordinates via $p = (x_1 + x_2)/2$ and $r = x_1 - x_2$ and analogously for p', r'. The integrals over r and r' are then trivially carried out because their integration range is $(-(\lambda+\lambda_c), \lambda+\lambda_c) \to (-\infty, \infty)$. In new coordinates $x = p - p'$ and $z = p + p'$ the result reads

$$\begin{aligned} I_f &= -\frac{1}{2} T^3 \int\limits_{-(\lambda+\lambda_c)}^{\lambda+\lambda_c} \frac{dx}{x} \mathrm{e}^{-x} \int\limits_{-2\lambda}^{2\lambda_c} dz \frac{x^2 - z^2}{\cosh x - \cosh z} \\ &= \frac{2\pi^2 T^3}{3} \int\limits_{-2(\lambda+\lambda_c)}^{2(\lambda+\lambda_c)} \frac{dt}{\mathrm{e}^t - 1} \left(1 + (\frac{t}{2\pi})^2\right) \\ &= -\frac{4\pi^2 T^3 (\lambda+\lambda_c)}{3} \left(1 + \frac{1}{3}\left(\frac{\lambda+\lambda_c}{\pi}\right)^2\right) \end{aligned} \tag{68}$$

where we have employed the integral (73). We see the explicit dependence of this off-shell integral on $T^3\lambda = T^2\epsilon_f$.

We would like to remark that the limitation of upper available energies is equivalent to cut- off procedures, which has been employed within memory collision integrals. This cut- off comes here from the approximate calculation of the collision integral and is fixed to reproduce the numerical results.

E A useful integral

To evaluate the useful integral

$$J_n = \mathrm{P} \int\limits_{-\infty}^{\infty} dx \frac{x^n}{\cosh x - \cosh a} \tag{69}$$

we use the integration contour as depicted in Fig. 7.

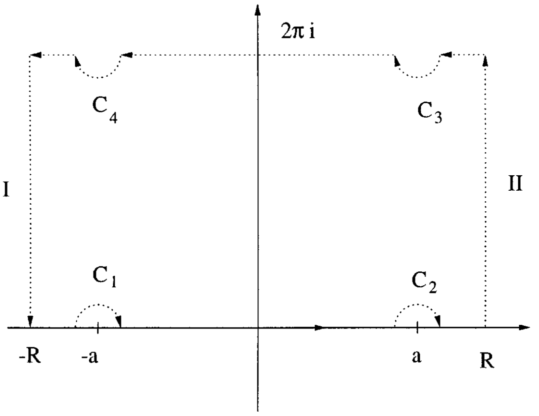

Fig. 7. The integration contour for the integral (73)

We have the following identity in the limit of large R and small radius around the poles

$$\mathrm{P} \int\limits_{-\infty}^{\infty} dx \frac{x^n - (x+2\pi i)^n}{\cosh x - \cosh a} = -c_1 - c_2 - c_3 - c_4 \tag{70}$$

where the c_i are the infinitesimal half circles surrounding the poles at $\pm a$. The latter ones are trivially calculated with the final result

$$\mathrm{P} \int\limits_{-\infty}^{\infty} dx \frac{x^n - (x+2\pi i)^n}{\cosh x - \cosh a} = \frac{i\pi}{\sinh a} \left(a^n - (-a)^n + (2\pi i + a)^n - (2\pi i - a)^n\right). \tag{71}$$

From this identity one now generates the required integrals J_n by calculating the identity (71) for $n+1$. The first two integrals read

$$\begin{aligned} J_0 &= -\frac{2a}{\sinh a} \\ J_2 &= -\frac{2a}{3\sinh a}(a^2 - 2\pi^2). \end{aligned} \tag{72}$$

As is obvious from symmetries only integrals J_n with even numbers of n are nonzero. The required integral in (68) reads now

$$\begin{aligned} \mathrm{P} \int\limits_{-\infty}^{\infty} dx \frac{x^2 - a^2}{\cosh x - \cosh a} &= J_2 - a^2 J_0 \\ &= \frac{4a}{3\sinh a}(\pi^2 + a^2). \end{aligned} \tag{73}$$

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